Lesson 30

FIND THE DISTANCE BETWEEN POINTS

NY-8.G.8

WORDS TO KNOW

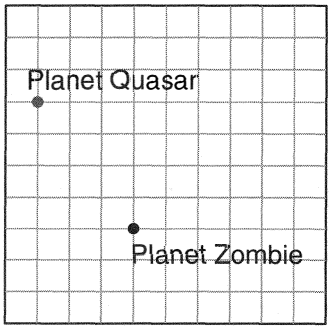
Pythagorean theorem

INTRODUCTION

Real-World Connection

Zelda is playing a video game in which she must move a game piece on a grid to collect valuable space rocks. For each unit that she moves, she uses up one coin. Zelda's current game screen is shown below, and her game piece is on Planet Zombie.

How many coins will Zelda use if she moves in a diagonal path from Planet Zombie to Planet Quasar? She can use the Pythagorean theorem to find out. Let's practice the skills in the **Guided Instruction** and **Independent Practice** and see how many coins Zelda uses at the end of the lesson!



What I Am Going to Learn

- How to use the Pythagorean theorem to find the distance between points on the coordinate plane

What I May Already Know

- I know how to find the distance between points on the coordinate plane that are in line horizontally or vertically, using coordinates.

Vocabulary in Action

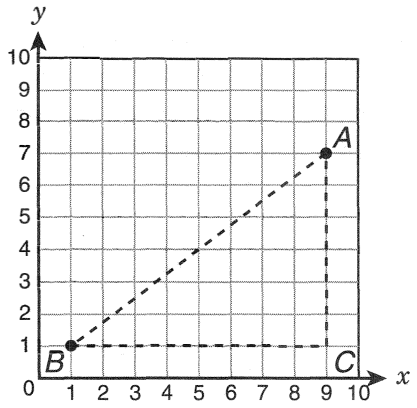
The **Pythagorean theorem** is used to find the distance between two points, *A* and *B*, on the coordinate plane.

- The line segment that would connect the points is the hypotenuse.
- To get from point *A* to point *B*, you would go a vertical distance and a horizontal distance.
- These lengths are the legs of a right triangle, and can be found from the coordinates of point *A* and *B*.
- Using the side lengths and the Pythagorean theorem, the distance between point *A* and point *B* can be found.

EXAMPLE

What is the distance between point *A* and point *B*?

Line segment *AB* is the hypotenuse of a right triangle.



Step One Find the length of *AC*, the vertical distance between *A* and *B*.

Counting on the grid, the length is 6 units.

Step Two Find the length of *BC*, the horizontal distance between *A* and *B*.

Counting on the grid, the length is 8 units.

Step Three Use the Pythagorean theorem to find the length of *AB*.

$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$10 = c$

Step Four Solve.

The distance from point *A* to point *B* is 10 units.

TURN AND TALK

Is there another different right triangle you could draw where line segment *AB* is the hypotenuse? Why?

You can also find the distance between any two points on the coordinate plane, even if you do not have the grid.

EXAMPLE

A line segment has endpoints at $A(-3, 2)$ and $B(4, 6)$. How long is the line segment?

Step One Find the horizontal side length. This is the difference between the x-coordinates: $4 - (-3) = 7$

Step Two Find the vertical side length. This is the difference between the y-coordinates: $6 - 2 = 4$

Step Three Apply the Pythagorean theorem.

$a^2 + b^2 = c^2$

$7^2 + 4^2 = c^2$

$49 + 16 = c^2$

$65 = c^2$

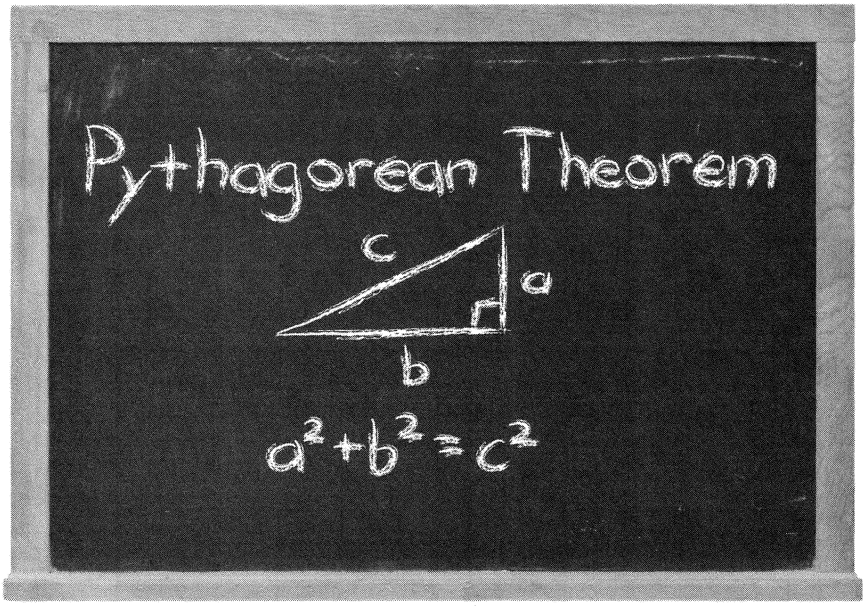
$\sqrt{65} = c$

$8 \times 8 = 64$, so $\sqrt{65}$ is a little more than 8.

So, line segment AB is a little more than 8 units long.

SKETCH IT

Make a quick sketch of the axes of the coordinate plane and the triangle if it helps to see the distance between the points horizontally and vertically.

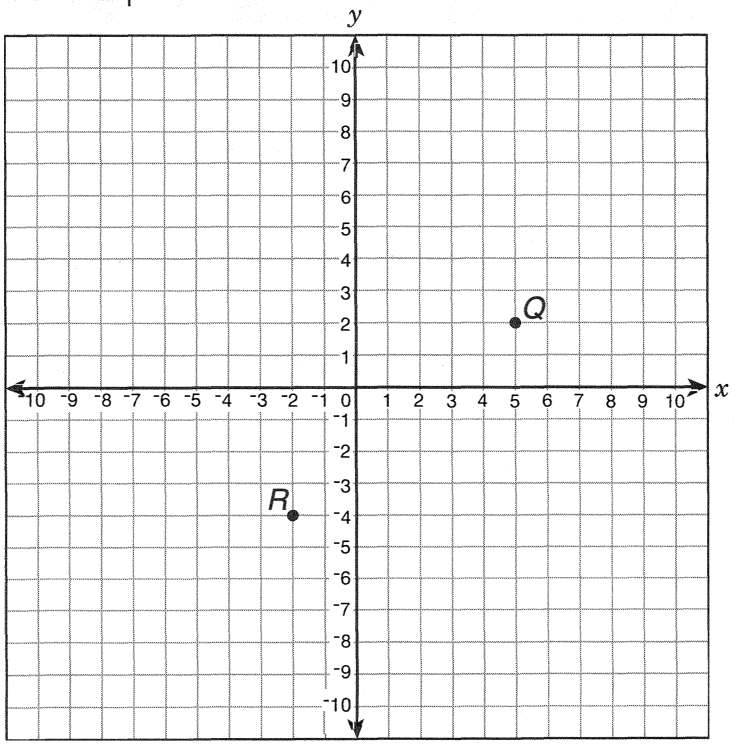


GUIDED INSTRUCTION

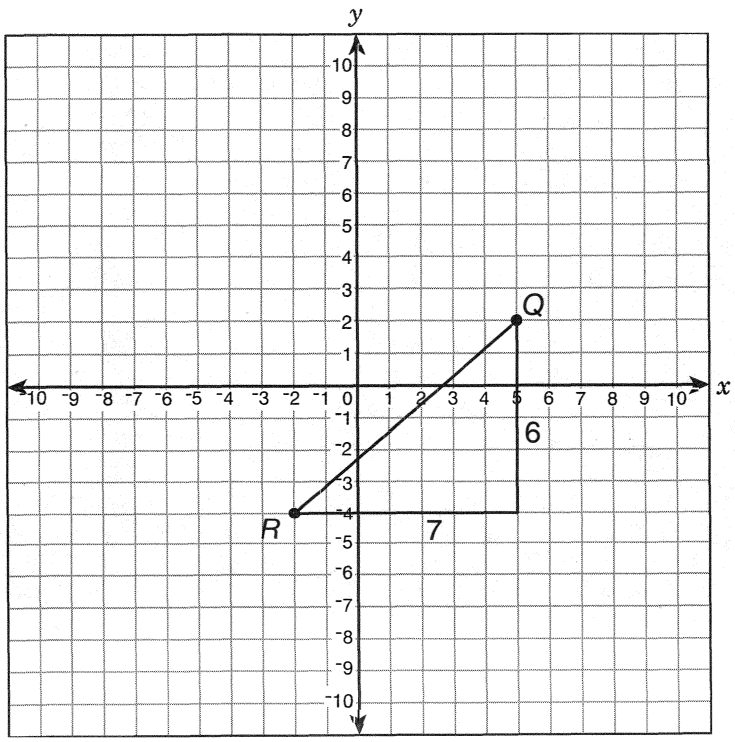
THINK ABOUT IT

You can estimate the distance between the points before using the Pythagorean theorem. Then you can decide if your answer is reasonable.

1. Two points, *Q* and *R*, are plotted on a coordinate plane. Find the distance between the points.



Step One Draw line segment *QR* and complete the right triangle.



Step Two Count the number of units to find the side lengths.

horizontal side = 7 units

vertical side = 6 units

Step Three Use the Pythagorean theorem to find the hypotenuse.

$a^2 + b^2 = c^2$

$7^2 + 6^2 = c^2$

$49 + 36 = c^2$

$85 = c^2$

$\sqrt{85} = c$

Step Four Solve.

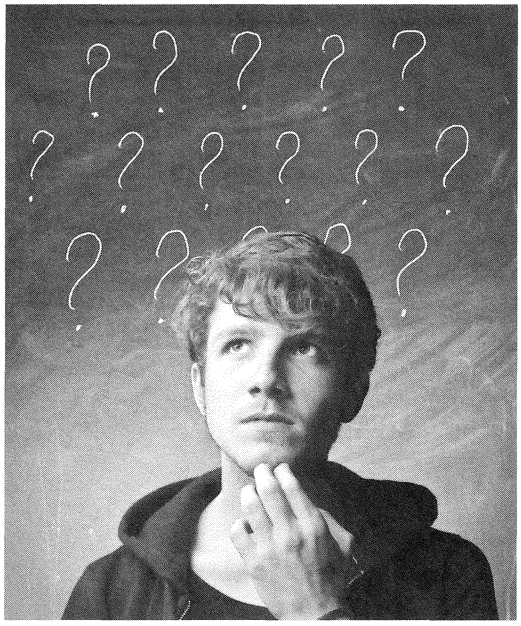
$9 \times 9 = 81$, so $\sqrt{85}$ is a little more than 9.

The distance between point Q and point R is a little more than 9 units.

SHARE IT

Can you think of an activity or game that you take part in with your family and that requires finding a distance that could be found using the Pythagorean theorem?

2. Two points, $E(6, 4)$ and $F(-2, 5)$, are plotted on a coordinate plane.



Find the distance between the points.

Step One Find the horizontal distance between the points.

Subtract the x-coordinates:

$6 - (-2) = 8$

Step Two Find the vertical distance between the points.

Subtract the y-coordinates:

$4 - 5 = -1$

Step Three Use the Pythagorean theorem to find the hypotenuse.

$a^2 + b^2 = c^2$

$8^2 + (-1)^2 = c^2$

$64 + 1 = c^2$

$65 = c^2$

$\sqrt{65} = c$

Step Four Solve.

$8 \times 8 = 64$, so $\sqrt{65}$ is a little more than 8.

The distance between point E and point F is a little more than 8 units.

TIPS AND HINTS

Notice that one point is the origin. What does this tell you about the sides of the right triangle?

3. Use your knowledge of the coordinate plane to find the distance between points (0, 0) and (12, 16). Select the number that correctly completes the statement.

The distance between points (0, 0) and (12, 16) is _____ units.

Ⓐ 14

Ⓑ 16

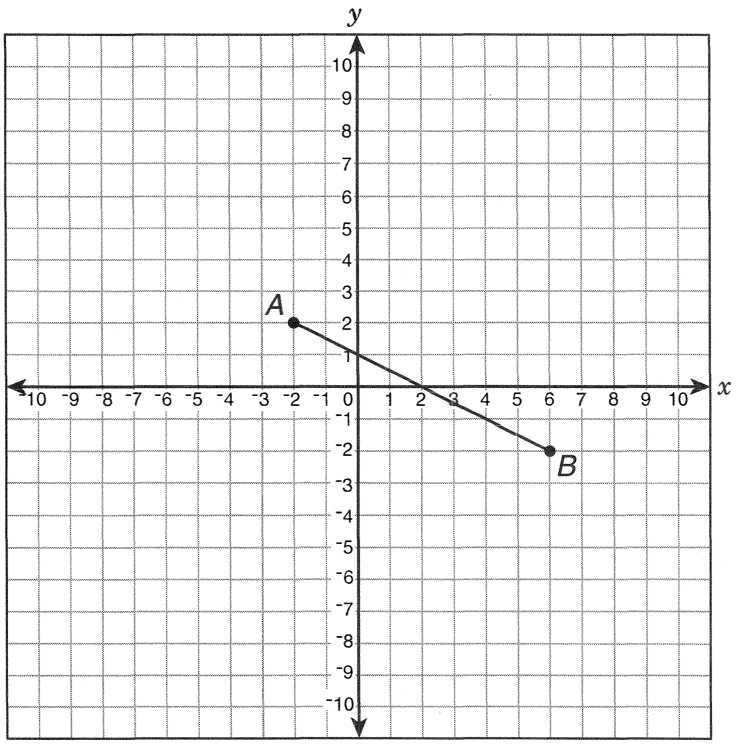
Ⓒ 20

Ⓓ 22

SKETCH IT

Draw the lengths of the vertical and horizontal sides if it helps visualize the right triangle.

4. Find the distance between points *A* and *B*.



The distance between the points is [ ] or about [ ] units.

Learning Together

Work with a partner. Using estimation and a coordinate grid, plot two points you think are about 9 units apart diagonally. Have your partner draw a right triangle and calculate the exact distance. Switch roles. Who came closer to 9 units?

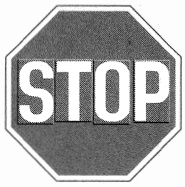
How Am I Doing?

What questions do you have?

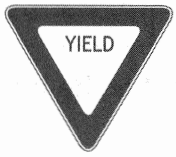
How can you find the distance between the origin and a point at (3, 4)?

If someone in your town asked for directions, could you direct them in a straight line, or would they need to travel in two directions?

Circle the sign that shows how you are doing with the skill.



I am stuck.

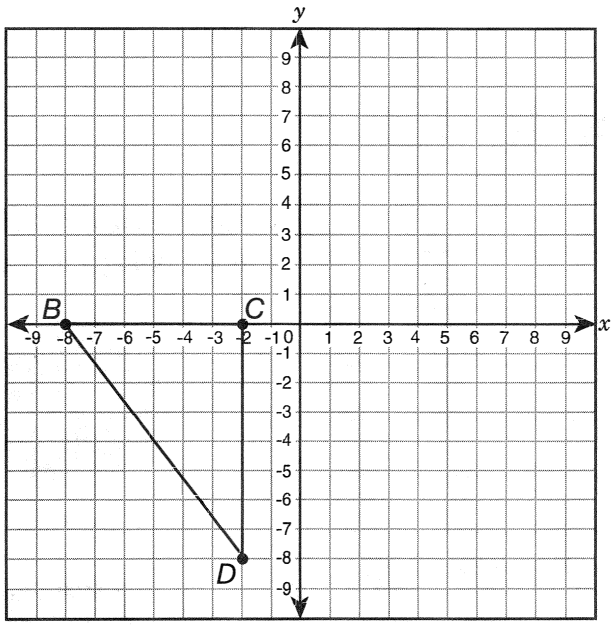


I almost have it.

I understand the skill.

INDEPENDENT PRACTICE 1

1 $\triangle BCD$ is shown on the graph below.



What is the length of side *BD*?

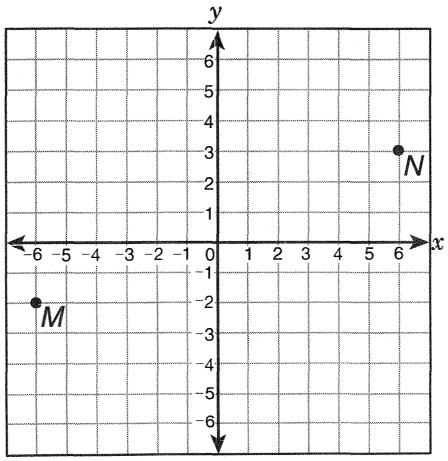
A 6

B 8

C 10

D 12

2 Two points are graphed on the coordinate plane below.



What is the distance, in units, between the two points?

A 10.9

B 13

C 14

D 18

THINK ABOUT IT

Why can't you simply count the spaces like you can for *BC* and *CD*?

TIPS AND HINTS

How can you find the distance between two points that are diagonal from each other on a coordinate plane?

3 Find the distance between (−3, −5) and (3, 4).

A $\sqrt{45}$

B $\sqrt{117}$

C 11

D $\sqrt{144}$

SKETCH IT

Plot the points on a coordinate plane so you have a visual.

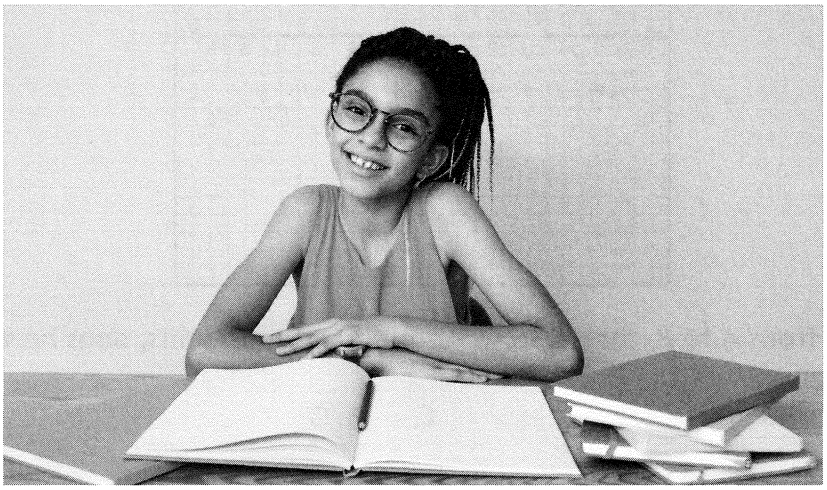
4 Two points on a map can be represented by (−4.5, 6) and (1, −5.2). Can you use the Pythagorean theorem to find the distance between the points?

Answer _____

Explain your answer.

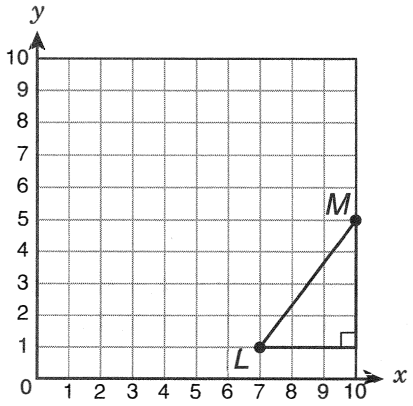
TIPS AND HINTS

Break the problem into smaller steps.



INDEPENDENT PRACTICE 2

1 A triangle is graphed on the coordinate plane below.



What is the distance, in units, between points *L* and *M*?

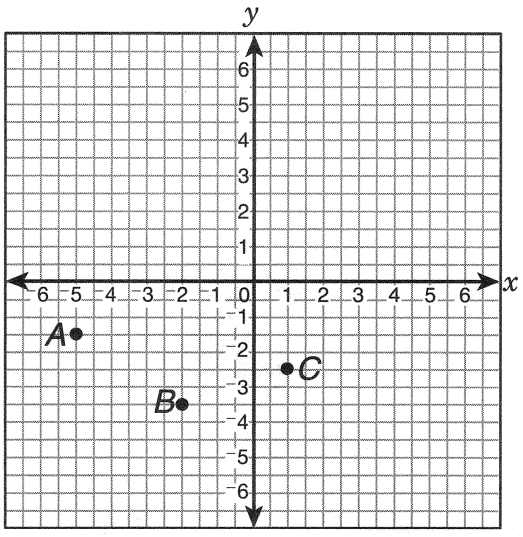
A 4

B 5

C 6

D 7

2 The three points shown on the coordinate grid below represent three locations on a map.



Hakeem walks from *A* to *B*, then *B* to *C*. About how far, in units, does he walk?

A 2.69

B 3.6

C 6.77

D 12.6

3 The table below shows locations of places on a map. Each grid space on the map represents 1 kilometer.

Place	Location
School	(2, 5)
Mall	(0, 9.8)
Post Office	(6, 11.1)

Patti drives from school to the post office, then back to the school. How far, in kilometers, does she drive?

A 7.3

B 10.1

C 14.6

D 20.2

4 What is the distance, in units, between points $O(-5, -2)$ and $P(7, -7)$, on a coordinate grid?

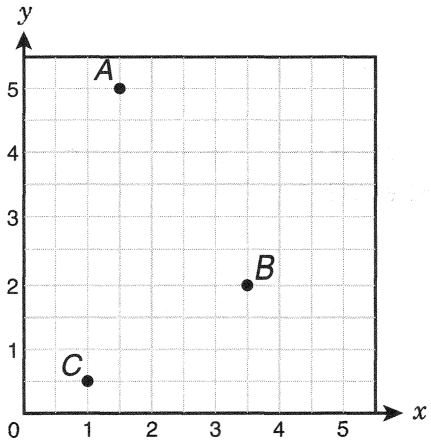
A 5

B 7

C 12

D 13

5 Three locations in Jacob's town are shown on the grid below.



Jacob walks from *A* to *B* and then to *C*. Which statement can be used to find how far he walks?

A $\sqrt{((5) - (2))^2 + ((1.5) - (3.5))^2} + \sqrt{((2) - (0.5))^2 + ((3.5) - (1))^2}$

B $\sqrt{((5) - (2)) + ((1.5) - (3.5))} + \sqrt{((2) - (0.5)) + ((3.5) - (1))}$

C $\sqrt{((5) + (2))^2 + ((1.5) + (3.5))^2} + \sqrt{((2) + (0.5))^2 + ((3.5) + (1))^2}$

D $\sqrt{((5) + (2)) + ((1.5) + (3.5))} + \sqrt{((2) + (0.5)) + ((3.5) + (1))}$

6 The points (18, −10) and (−6, *y*) are graphed on a coordinate plane. If the distance between the points is 25 units, what could be the value of *y*?

A 9

B −3

C −11

D −16

7 Billy is flying his new radio-controlled helicopter around town. He is using a map in which each grid line is equivalent to 100 feet. Billy releases the helicopter from the library parking lot, at (2, 6) on the map. He gets it to cruising altitude and then starts measuring its flight. Billy flies the helicopter in a direct line to the town pool, at (6, 9) on the map. How far, in feet, has the helicopter flown?

A 400

B 500

C 600

D 700

8 Write an expression that correctly uses the Pythagorean theorem to find the distance between *A*(−8, −8) and *B*(10, 7).

Answer _____

Explain your answer.

_____

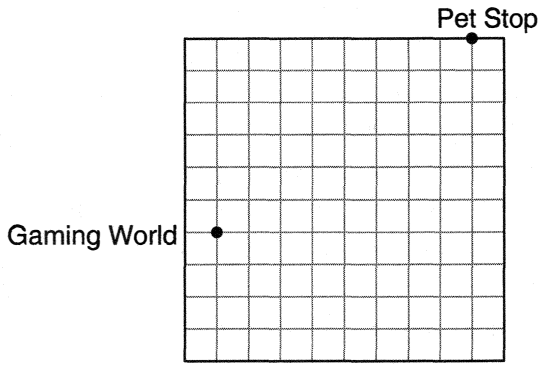
_____

_____

_____

_____

9 The diagram below shows a map of two stores in a mall. Each unit on the map represents 10 yards.



What is the distance between Pet Stop and Gaming World in yards?

Answer _____ yards

Explain your answer.

_____

_____

_____

_____

_____

A new owner wants to open her game store at point (5, 1). However, the store must be at least 60 yards from Gaming World. Can the owner open at this location?

Answer _____

Explain your answer.

_____

_____

_____

_____

_____

EXIT TICKET

NY-8.G.8

Now that you have mastered finding the distance between points, let's solve the problem in the **Real-World Connection.**

Zelda is playing a video game in which she must move a game piece on a grid to collect valuable space rocks. For each unit that she moves, she uses up one coin. Zelda's current game screen is shown below, and her game piece is on Planet Zombie.

How many coins will Zelda use if she moves in a diagonal path from Planet Zombie to Planet Quasar?